

Exercise 21

Find the derivative of the function.

$$y = \sqrt{\frac{x}{x+1}}$$

Solution

Take the derivative using the quotient rule and the chain rule.

$$\begin{aligned} y' &= \frac{dy}{dx} = \frac{d}{dx} \left[\left(\frac{x}{x+1} \right)^{1/2} \right] \\ &= \frac{1}{2} \left(\frac{x}{x+1} \right)^{-1/2} \cdot \frac{d}{dx} \left(\frac{x}{x+1} \right) \\ &= \frac{1}{2} \left(\frac{x}{x+1} \right)^{-1/2} \cdot \frac{\left[\frac{d}{dx}(x) \right] (x+1) - \left[\frac{d}{dx}(x+1) \right] (x)}{(x+1)^2} \\ &= \frac{1}{2} \left(\frac{x}{x+1} \right)^{-1/2} \cdot \frac{(1)(x+1) - (1)(x)}{(x+1)^2} \\ &= \frac{1}{2} \left(\frac{x}{x+1} \right)^{-1/2} \cdot \frac{1}{(x+1)^2} \\ &= \frac{1}{2x^{1/2}(x+1)^{3/2}} \end{aligned}$$